On the passibility of a short-lived free Dirac particle

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ABSTRACT

A time-decaying Dirac equation is suggested. For a free particle with rest mass about 0.5GeV, the live-time is about 10^{-25} second in the rest frame.

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In two recent articles [1], the author has discussed some properties concerning a free Dirac electron. Based on Dirac equation, a possibility of a complex 4-dimensional spac-time manifold was suggested, which leads to a function for describing a free particle should be a free wave. Furthermore since there are couplings between the spatial variables and the internal variables $(\alpha_i p_i)$ [2] in the Hamiltonian, the generalized coordinates become mixtures of spatial coordinates and the internal coordinates. In fact, Dirac [2] calculated that $x_1 = a_1 + c^2 p_1 H^{-1} t + \frac{i}{2} c\hbar(\alpha_1 - cp_1 H^{-1})H^{-1}$. As a consequence the velocities of the genrealized coordinates $\dot{x}_i's$ are $c\alpha_i$. However, the velocities of the spatial coordinates $v_i = p_i H^{-1} (i = 1, 2, 3)$ which commute with H and can be simultaneously measured with H. That is, there two kinds of velocities for a free Dirac particle. In this note, it is suggested that a short-lived free Dirac particle may exist which may be described by a time decaying Dirac equation.

Given a Hamiltonian

$$H = c(p_x^2 + p_y^2 + p_z^2 + m^2 c^2)^{1/2}. \tag{1}$$

Let us define a Dirac process as the following: Introduce the $\alpha_i's$ and β together with their relations as defined in ref.[2]. Then take the square root of the right-hand-side of equation (1), we obtain the Dirac Hamiltonian

$$H = c\boldsymbol{\alpha} \cdot \boldsymbol{p} + mc^2 \beta. \tag{2}$$

In this way, the internal variables are introduced. Meanwhile, interactions between the spectial variables and the internal variables are also introduced. The generalized coordinales are created and become mixtures of the spatial varibles and the internal

variables. Also, two kinds of velocities are produced.

In classical machanics, a Hamiltonian $H(p,q)$ is defined for a free particle as

$$H(p,q) = \sum p_i \dot{q}_i - L(q_i, \dot{q}_i), \qquad (3)$$

where the $q_i's, \dot{q}_i's$ and the $p_i's$ are the generalized coordinates, generalized velocities and genrealized momenta. The $\dot{q}_i's$ are supposed to be eliminated by some available relations. For a relativistic free particle, L is given as [3]

$$L = -mc^2(1 - \frac{1}{c^2} \sum \dot{q}_i^2)^{1/2}. \qquad (4)$$

If we substitude equation (4) into equation (3) and elimmate the $\dot{q}_i s$ by $\dot{q}_i = p_i H^{-1}$, we will have equation (1). Then we can perform a Dirac process to obtain Dirac equation. However, if we perform the Dirac process one step before we get to equation (1). That is, we perform the Dirac process for the function L of equation (4) and substitute into equation (3), then we have

$$H(p,q) = \sum p_i \dot{q}_i + mc^2(\beta \pm \frac{i}{c} \sum \alpha_i \dot{q}_i), \qquad (5)$$

where the $\pm$ sign is determined by the condition for a finite solution with respect to the forward or backword propergating wave. Now we have a Dirac type of Hamiltonian where the $\dot{q}_i's$ are to be eliminated by suitable relations. If we tak $\dot{q}_i = p_i H^{-1}$, it is straightforward to show that equation (5) reduces to equation (2) by noticing that, $H^2 - c^2 p^2 = (mc^2)^2$, and $(-i\beta\alpha_i's, \beta)$ is equivalent to $(\alpha_i's, \beta)$. We are aware that the Hamiltonian (5) is supposed to describe a free Dirac particle, if this is correct, then a Dirac free particle has two kinds of velocities, velocities of the generalized coordinates

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and those of the spatial coordinates. Therefore we can also take $\dot{q}_i = c\alpha_i$ in equation (5). Thus we obtain

$$H = \sum c\alpha_i p_i + mc^2\beta \pm 3imc^2. \qquad (6)$$

This Hamiltonian will result with a solution having a couplex energy that represents a particle with a life-time about $\hbar/(3mc^2)$. For a particle with rest mass about 0.5 GeV, the life-time is about 10^{-25} second in the rest frame. In the lab-frame, its life-time can be determined by relativity.

We conclude that a Lagrange-Hamilton formalism may lead to two kinds of Dirac particles, one kind of them are stable particles, the other unstable. Finally, one may wish to ask whether one of those un-discovered particles, such as a quark, may belong to the second kind of particles. If an unstable particle of this kind would be created from a source, it can only live for such a short time, then decay immediately. If there is no available exit channel, it must go back to the source again.

References

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